COLD DAMPING IN MICROSCOPE

A QUANTUM THERMODYNAMICAL ANALYSIS OF NOISE

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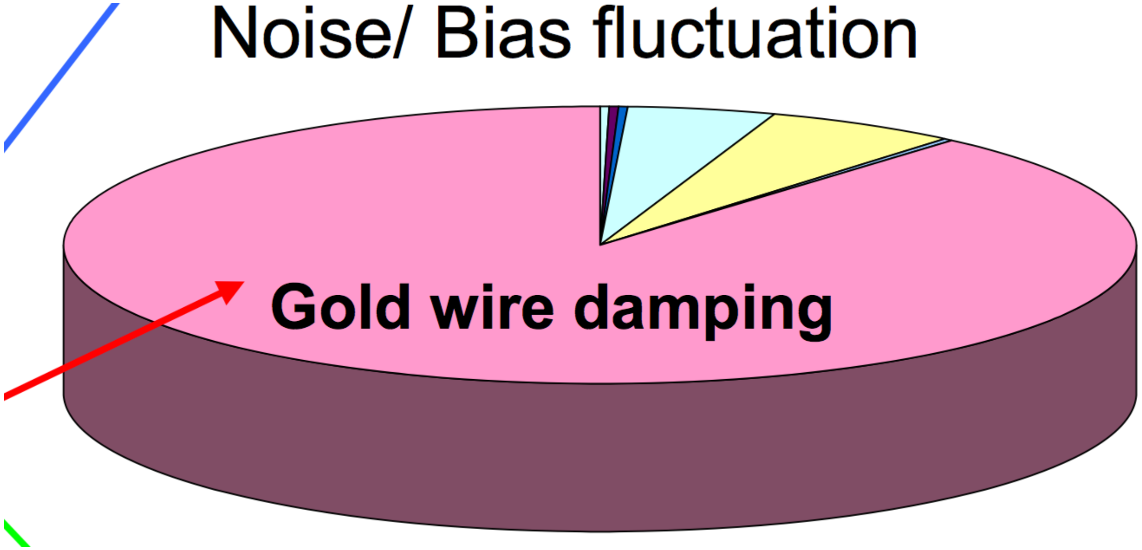
MICOSCOPE COLLOQUIUM III

PALAISEAU, 4 NOV 2014

Noise/ Bias fluctuation



Total noise = 4.89 $10^{-12} ms^{-2}/Hz^{1/2}$	Gold wire damping noise = 4.85 $10^{-12} ms^{-2}/Hz^{1/2}$

Main noise source : thermal noise of mechanical damping due to 7 μm gold wire

Active damping from servo control does not increase noise

OUTLINE

The physics of cold damping

- analysis in term of thermodynamics
- ultimate limits and compatibility with quantum fluctuations

Using cold damping to improve measurements

- early modelization of a capacitive accelerometer

Proposal for Microscope

COLD DAMPING

Cold damping : Active reduction of thermal noise in electro- or opto-mechanical systems.

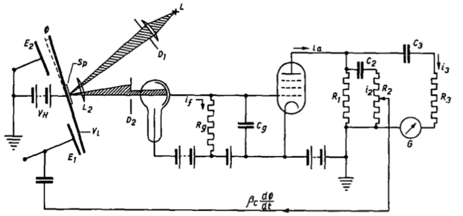
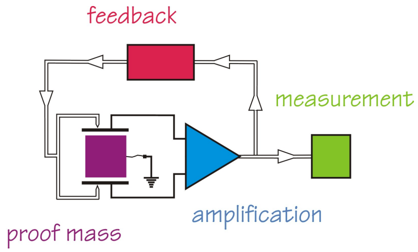
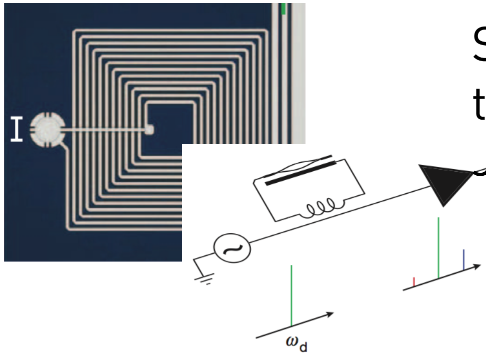


Fig. 1. Schematic arrangement of the electrometer, the damping circuit and the recording galvanometer.

The reduction in the brownian motion of electrometers,
J.M.W. Milatz et al. Physica XIX, 195-207 (1953)

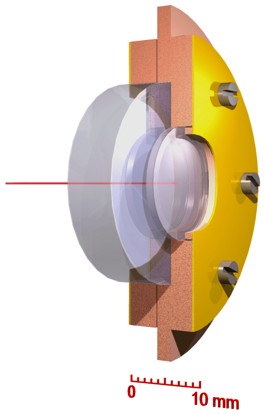


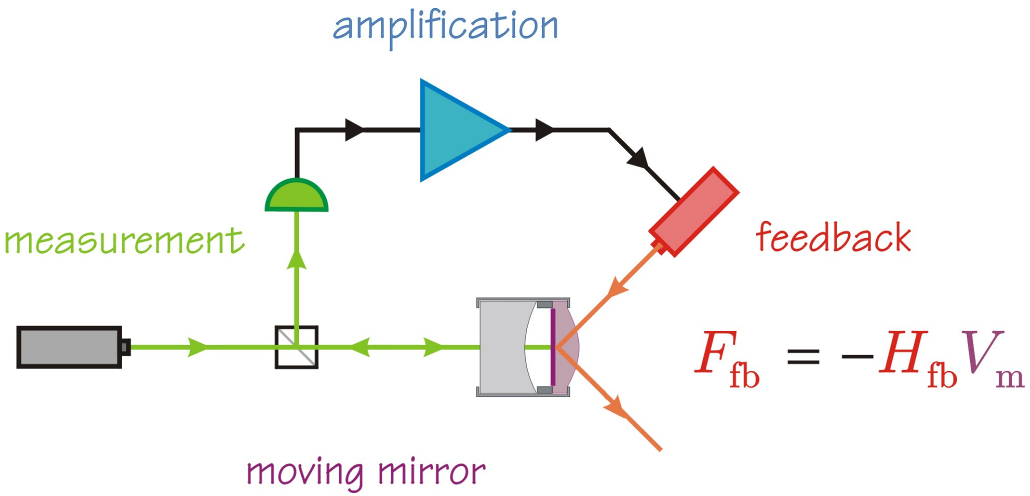
Quantum theory of fluctuations in a cold damped accelerometer.
F. Grassia, J.M. Courty, S. Reynaud, P. Touboul.
European Physical Journal D, 2000, 8, pp.101-110

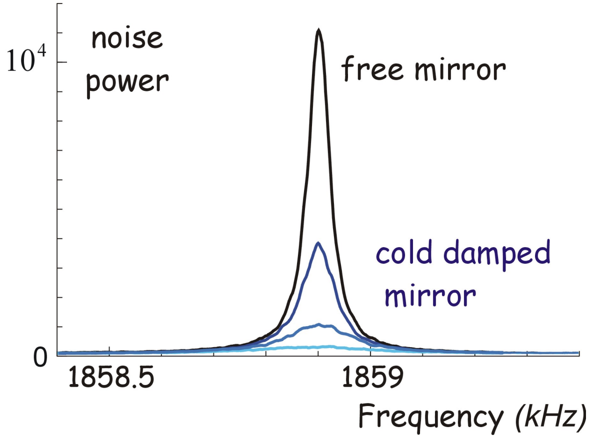


Sideband cooling of micromechanical motion
to the quantum ground state
J. D. Teufel et al. Nature 475, 359–363 (21 July 2011)

COLD DAMPING IN OPTOMECHANICAL SYSTEM







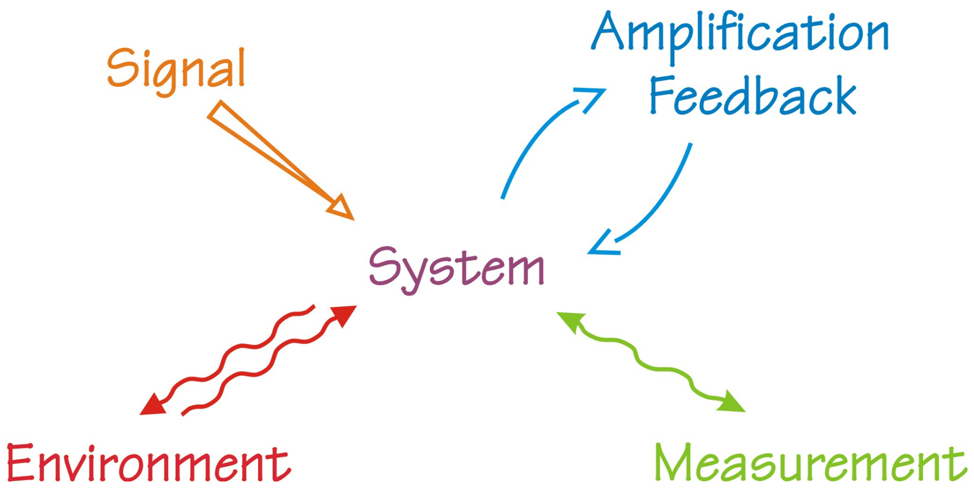
Mechanical oscillator is
an acoustic mode of a miror

Motion is measured with Fabry Perot

Force is radiation pressure of a laser

Cooling of a mirror by radiation pressure
P.F. Cohadon, A. Heidmann, M.Pinard, Phys. Rev. Lett. 83, 3174 (1999).

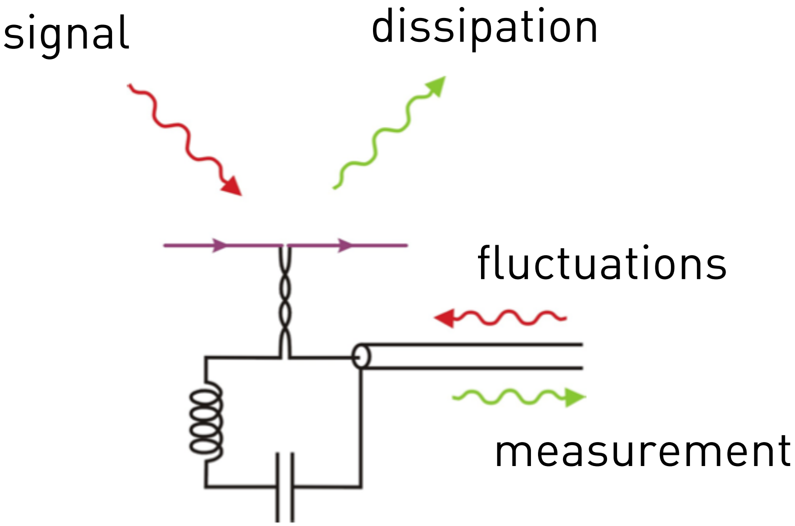
NON IDEAL MEASUREMENTS



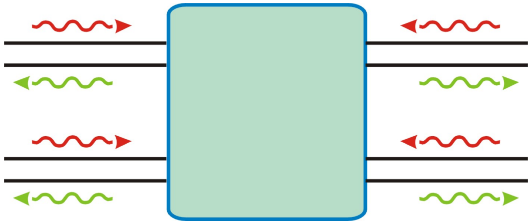
Important elements to consider in sensitivity analysis of actual measurements :

- Thermodynamical and quantum noises
- Active systems for signal amplification and servocontrols
- Spectral analysis of noise
- Modelization of complex devices

MEASUREMENT AS A SCATTERING



Fluctuation-dissipation theorem ensures consistency between the thermal noise of the oscillator and the coupled fluctuations bath



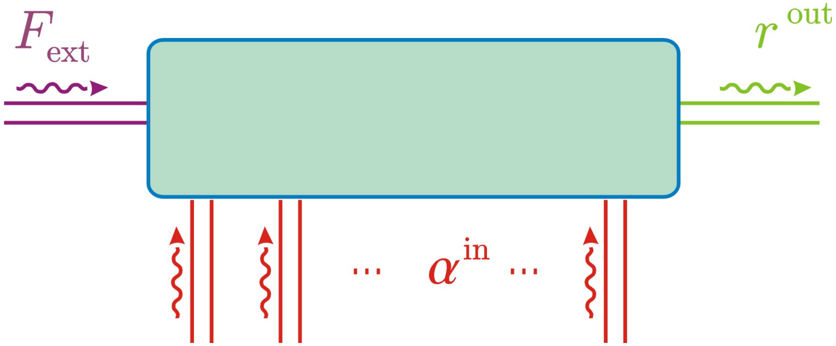
Unitarity of S matrix enforces

thermodynamic constraints

quantum constraints

Scattering of quantum fields

DESCRIPTION OF A MEASUREMENT



Force estimator

$$\hat{F}_{\text{ext}} \propto r^{\text{out}}$$

$$= F_{\text{ext}} + \sum_{\alpha} \mu_{\alpha} a^{\text{in}}$$

Added noise

$$\Sigma_{FF} = \sum_{\alpha} \left|\mu_{\alpha}\right|^{2} \sigma^{\text{in}}_{\alpha\alpha}$$

Thermal and quantum noises

$$\sigma^{\text{in}}_{aa}[\omega] = \frac{1}{2} \coth \frac{\hbar|\omega|}{k_B T_a}$$

$$= \frac{1}{e^{\frac{\hbar|\omega|}{k_B T_a}} - 1} + \frac{1}{2}$$

Thermal

Quantum

$$T \to \infty \qquad \hbar\,|\omega|\,\sigma_{aa} \simeq k_B T_a$$

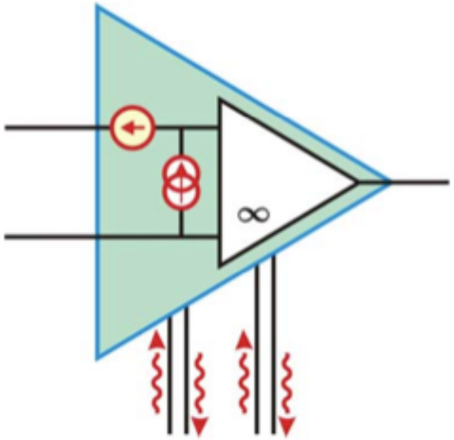
$$T \to 0 \qquad \hbar\,|\omega|\,\sigma_{aa} \simeq \frac{1}{2}\hbar\,|\omega|$$

NOISE IN ACTIVE SYSTEMS

Amplification is also treated as a scattering.

Example : operational amplifier

Voltage noise and current noise

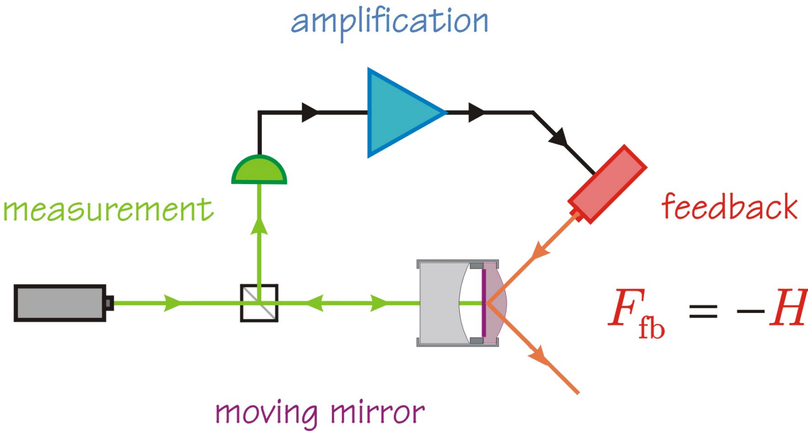


Characterized by noise impedance and noise temperature

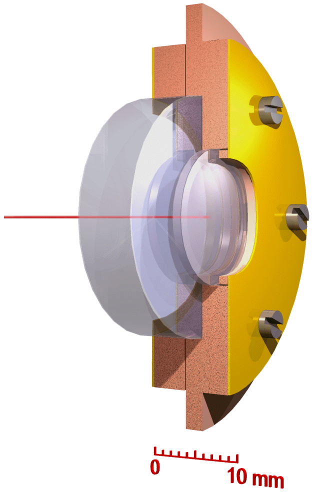
Charge and Flux are conjugated in quantum regime.

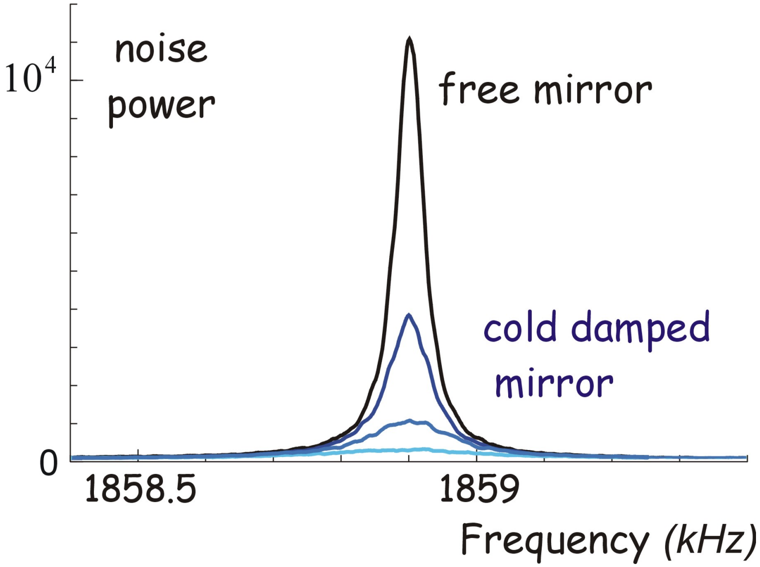
Quantum noise in ideal operational amplifiers,
Courty, Grassia, Reynaud, Europhys. Lett. 46 (1999), 31-37

COLD DAMPING IN OPTOMECHANICAL SYSTEM



$$F_{\mathrm{fb}} = -H_{\mathrm{fb}} V_{\mathrm{m}}$$



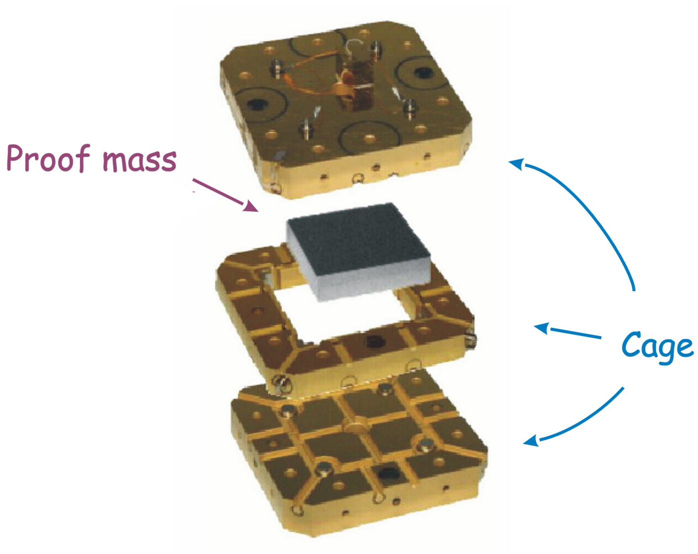


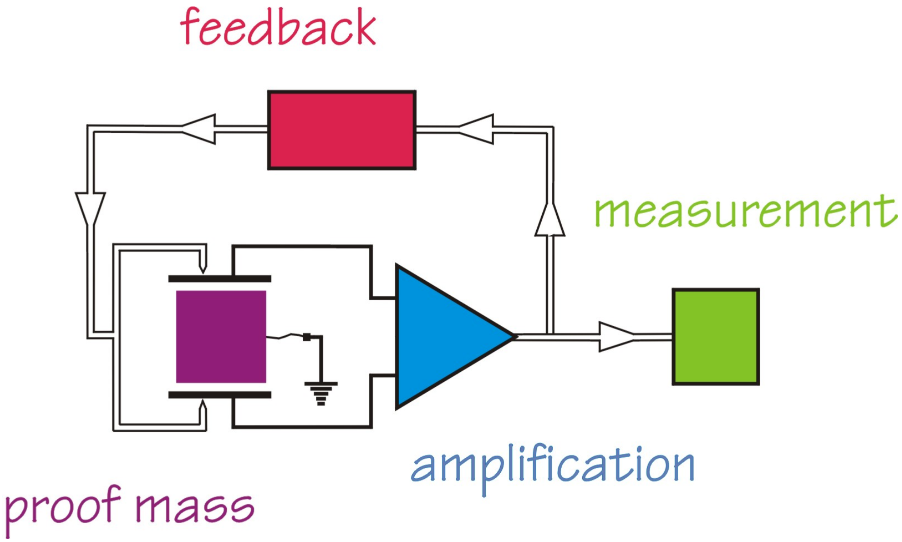
$$T = \frac{H_m}{H_m + H_{fb}} T_m$$

Cooling of a mirror by radiation pressure

P.F. Cohadon, A. Heidmann, M.Pinard, Phys. Rev. Lett. 83, 3174 (1999).

THE STAR ACCELEROMETER





Detection
    amplification noise
    back action of servocontrol

Active control of proof mass
    restoring force
    cold damping

$$\sqrt{\Sigma_{aa}} = 1.2 \cdot 10^{-12}\, m\, s^{-2} \,/\, \sqrt{Hz}$$

Quantum theory of fluctuations in a cold damped accelerometer.
F. Grassia, J.M. Courty, S. Reynaud, P. Touboul. E.P.J.l D, 2000, 8, pp.101-110

$$\widehat{F}_{ext} = F_{ext} + \sum_{\alpha} \mu_\alpha \alpha^{\text{in}}$$

Characterization of all the measurement characteristics
measurement, noise, backaction, correlations

$$\begin{aligned}
\mu_m &= -\sqrt{2\hbar\,|\Omega|\,H_m} \\
\mu_{l_2} &= -\frac{i\Omega\sqrt{\hbar}}{\sqrt{2R_l\omega_t\varkappa_t}}\Xi_m \qquad \mu_{l_1} = 0 \\
\mu_{r_1} &= -\frac{\Omega\sqrt{\hbar R_r}}{2\sqrt{2\omega_t}Z_f\varkappa_t}\Xi_m \qquad \mu_{r_2} = 0 \\
\mu_{a_1} &= -\mu_{b_1} = \sqrt{2\hbar R_a\omega_t}\left(-\varkappa_t + \frac{\Omega}{2\varkappa_t\omega_t Z_f}\Xi_m\right) \\
\mu_{a_2} &= -\frac{i\Omega\sqrt{\hbar R_a}}{\sqrt{2}\varkappa_t\sqrt{\omega_t}}\Xi_m\left(\frac{1}{R_a} - \frac{1}{R_l} - \frac{1}{Z_t}\right) \\
\mu_{b_2} &= -\frac{i\Omega\sqrt{\hbar R_a}}{\sqrt{2}\varkappa_t\sqrt{\omega_t}}\Xi_m\left(\frac{1}{R_a} + \frac{1}{R_l} + \frac{1}{Z_t}\right)
\end{aligned}$$

Noise of active control negligeable
low temperature of amplifier noise
ratio of signal frequency and operation frequency

$$\Xi_m = H_m - iM\Omega + \frac{iK}{\Omega}$$

Sensitivity limited by residual damping

$$\Sigma_{FF} = 2\;H_m\;k_B\;\Theta_m$$

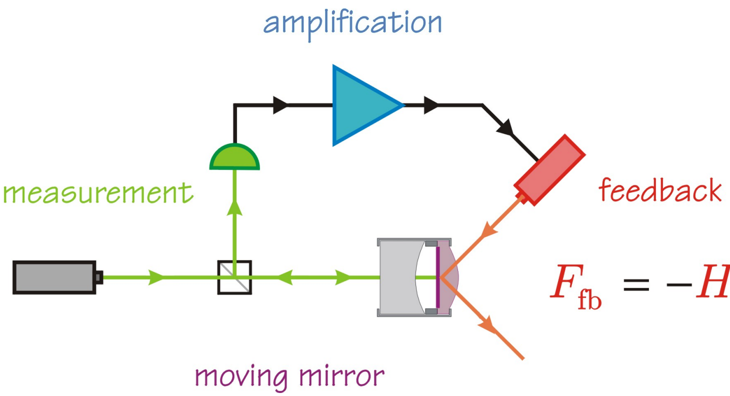
$$+4\left(r + \frac{1}{r}\right)|\Xi_m|\,\frac{\Omega}{\omega_t}\;k_B\Theta_a$$

$$r = 2\;\varkappa_t^2\;\frac{\omega_t}{\Omega}\;\frac{R_a}{|\Xi_m|}$$

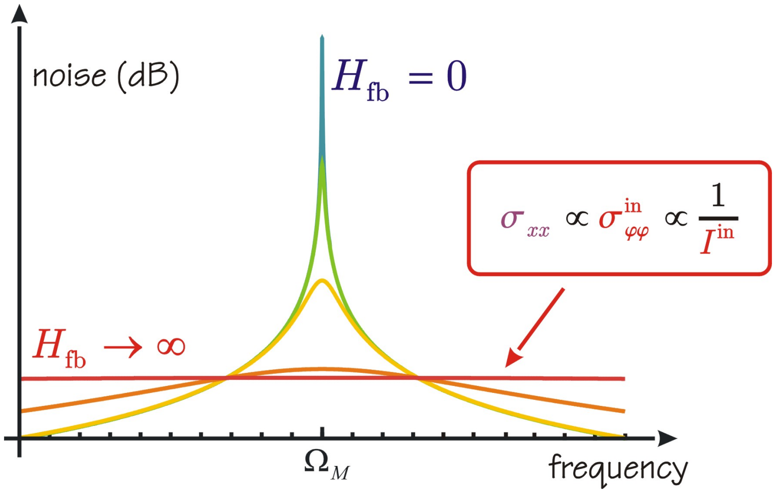
$$\begin{aligned}
M &= 0.27\;kg \\
H_m &= 1.3\cdot 10^{-5}\;kg\;s^{-1} \\
\Theta_m &= 300\;K \\
\Omega &= 2\pi\cdot 5\cdot 10^{-4}\;Hz \\
\omega_t &= 2\pi\cdot 10^{5}\;Hz \\
R_a &= 1.5\cdot 10^{5}\;\Omega \\
T_a &= 1.5\;K
\end{aligned}$$

Quantum theory of fluctuations in a cold damped accelerometer.
F. Grassia, J.M. Courty, S. Reynaud, P. Touboul. E.P.J.l D, 2000, 8, pp.101-110

QUANTUM LIMIT OF COLD DAMPING



$$F_{\mathrm{fb}} = -H_{\mathrm{fb}} V_{\mathrm{m}}$$



$$E = \hbar\Omega_m \left(n_T \frac{H_m}{H_m + H_{fb}} + \frac{1}{2} \right)$$

Energy is reduced to ground state energy -↑ temperature is zero

J.M. Courty, A. Heidmann, M. Pinard Eur. Phys. J. D 17, 399 (2001)

PROPOSAL

Response to the call for ideas

Collaboration with Microscope Team in order to :

Perform the analysis on current sensor design

Evaluate effectiveness of cold damping

Use noise as a source of information on the instrument operation physics