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LISAPF Flight Through the Saddle Point

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Introduction

- Attempt to look at possible LISAPF constraints on Newtonian gravity in the solar system
- Very crude limits of validity of Kepler's Third Law have been determined in the acceleration range between 7.8 ms^{-2} and 1.3×10^{-6}ms^{-2} assuming simple 2-body dynamics.
- More refined limits have been determined from results of numerical integrations of orbits of solar system bodies by Pitjeva in 2005
- Using a model-independent approach the potential for LISAPF to make sensitive gradiometer measurements to extend the range of validity and precision has been assessed.
- The acceleration range can be extended down to 1×10^{-7}ms^{-2}.
- The precision of validation at the low acceleration range (1×10^{-7}ms^{-2} to 5×10^{-5}ms^{-2}) can potentially be marginally improved.
- The extended range corresponds effectively to three times the current size of the solar system.

Kepler's Third Law

Newtonian Gravity between two bodies, M_1 and M_2

$$\frac{a^3}{P^2} = \frac{G(M_1+M_2)}{4\pi^2}$$

a - is semi-major axis

P – is orbital period

Add non-specific term to force

$$F = \frac{GM_1M_2}{r^2} + \delta F$$

$$\frac{a^3}{P^2} = \frac{G(M_1+M_2)}{4\pi^2} + \frac{\delta F r^2}{M_2}$$ assumes $M_2 \ll M_1$

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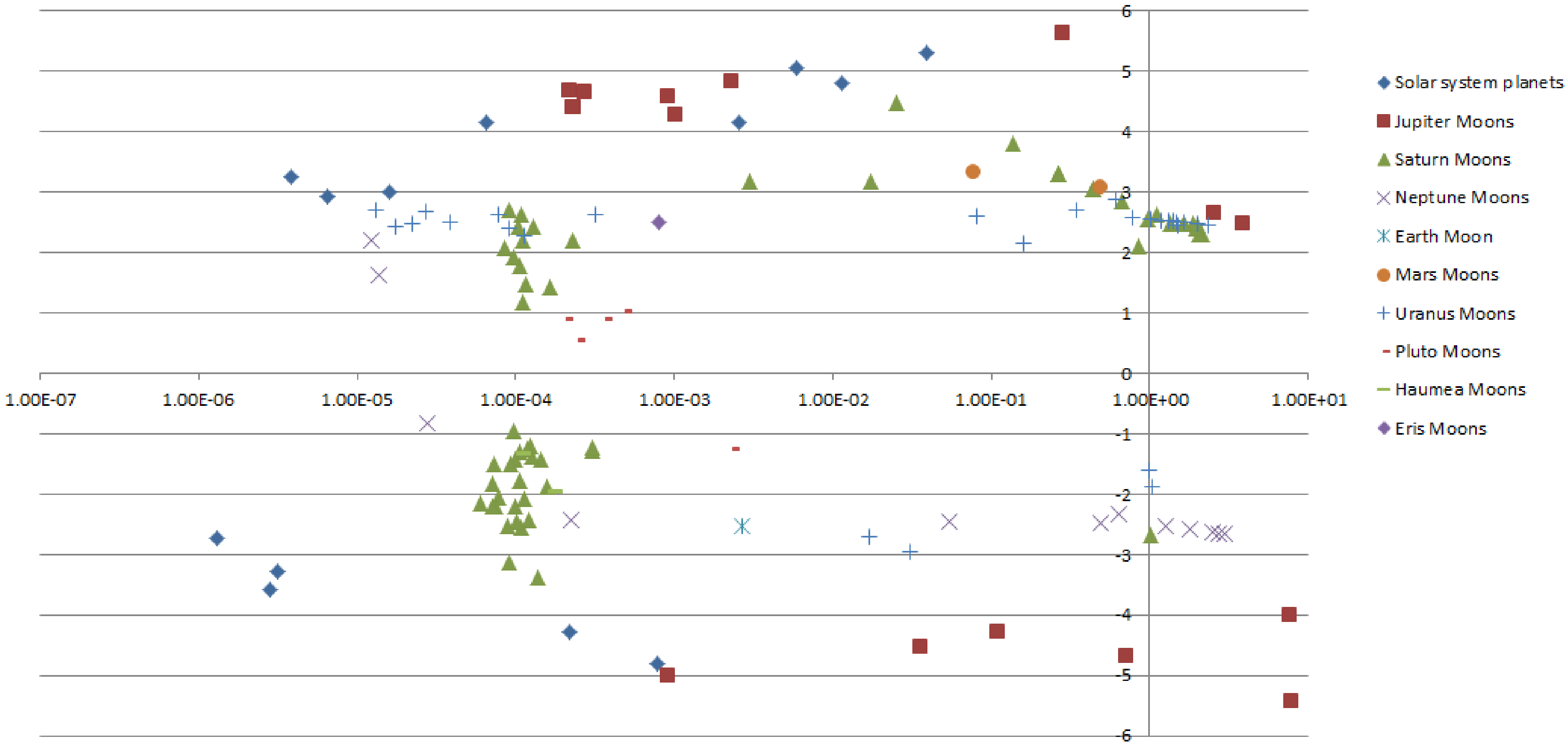
Then

'Residual' $= 1 - \left(\frac{a^3}{P^2} / \frac{G(M_1+M_2)}{4\pi^2} \right) = \frac{\delta F}{F}$

- Plotted residual for all solar system planets and dwarf planets in orbit around the Sun
- Plotted residuals for all satellites around other bodies
- Used log plot to expand range
- Kept sign to look for statistical quality

Residuals plot



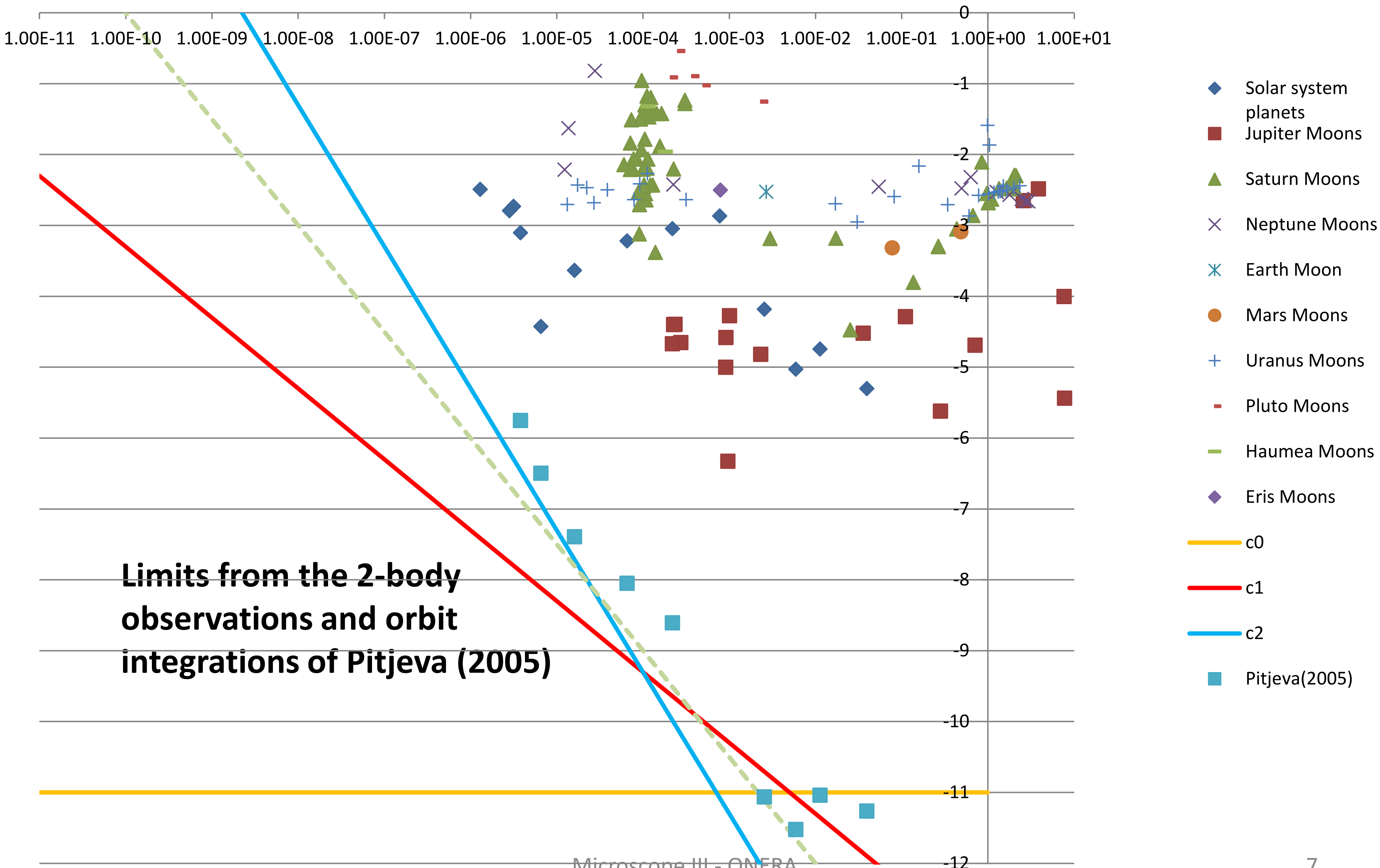
Y=sign(res)*log10(res)

X=local acceleration ms^{-2}

Discussion of Residuals

- Solar system is multi-body object itself in orbit within Galaxy.
- Numerical orbit integration provides best way of allowing for all interactions.
- Pitjeva (2005) – 317,000 position observables (1913-2003) to get
  - 1AU = $149597870.6960 \pm 0.0001$ km (7 in 10^{13})
  - 9 planets + Sun + Moon residuals to semi-major axes quoted
  - PPN formulism includes GR

Assume $\frac{\delta a}{a} = \sum c_n a^{-n}$



Motivation

- **LISA Pathfinder offers the following measurement capability (ESA-SCI(2007)1):**
  - **Differential Force Measurement Sensitivity:**

    $\approx 1.3\text{x}10^{-14}\text{N} / \sqrt{\text{Hz}}$ around 1mHz
  - **Drag-Free Platform Stability:**

    Platform Free-Fall Quality of

    $\approx 10^{-13}\text{ms}^{-2}/ \sqrt{\text{Hz}}$ around 1mHz

    $\approx 10^{-9}\text{ms}^{-2}$ at DC
  - **Gravity Gradiometer Sensitivity:**

    $\approx 1.5\text{x}10^{-14}\text{s}^{-2}/ \sqrt{\text{Hz}}$ around 1mHz

Potential LISAPF Limits

LISAPF measures Δa at a particular location in space, s.

The ‘additional’ signal due to δF is

$$\Delta a = \frac{\partial(\delta a)}{\partial a}\left(\frac{\partial a}{\partial r}\right)_s \Delta r$$

where Δr is separation between proof-masses

Potential LISAPF Limits

$$\Delta a = \frac{\partial(\delta a)}{\partial a}\left(\frac{\partial a}{\partial r}\right)_s \Delta r$$

where Δr is separation between proof-masses

and $\left(\frac{\partial a}{\partial r}\right)_s = \frac{2a_E}{r_E}$ where a_E is gravitational acceleration on the spacecraft and r_E is distance between spacecraft and Earth

Potential LISAPF Limits

$$\Delta a = \frac{\partial(\delta a)}{\partial a}\left(\frac{2a_E}{r_E}\right)\Delta r$$

Recalling the assumption $\frac{\delta a}{a} = \sum c_n a^{-n}$ gives

$$\Delta a = \sum (n-1) c_n a^{-n} \left(\frac{2a_E}{r_E}\right)\Delta r$$

Limits on c_n from assuming limiting sensitivity to Δa

Potential LISAPF Limits

Some numbers used

$\left(\frac{2a_E}{r_E}\right) = 4.59 \times 10^{-11}$ms^{-2}/m $\Delta r = 0.4$m

$\Delta a_{min} = \frac{3\times10^{-14}}{\sqrt{1000}}$ ms^{-2} for 1000s integration

Then $c_{0_{min}} = \frac{\Delta a_{min}}{(4.59\times10^{-11}\times0.4)} = 5 \times 10^{-5}$

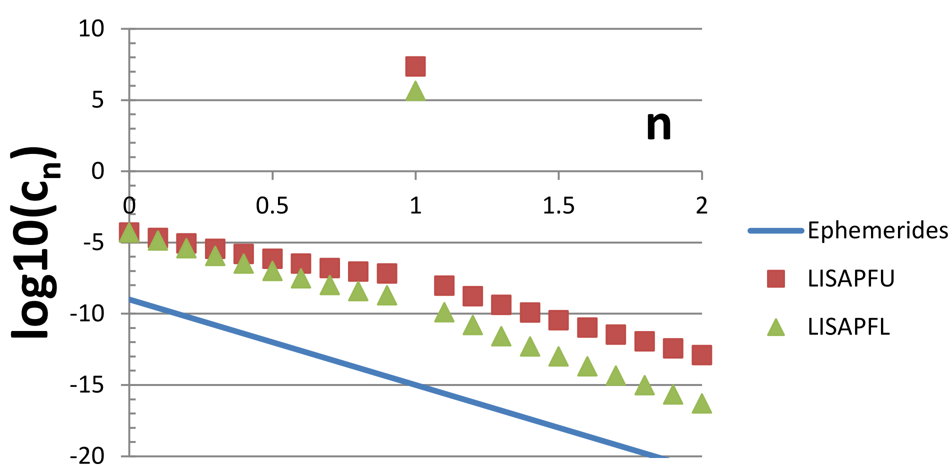
$$c_{1_{min}} = \infty$$

$$c_{2_{min}} = \frac{\Delta a_{min} a^2}{(4.59 \times 10^{-11} \times 0.4)}$$

Potential LISAPF Limits

Make it more general by allowing n to be non-integer

$$c_{n_{min}} = \frac{\Delta a_{min} a^n}{(n-1)(4.59 \times 10^{-11} \times 0.4)}$$

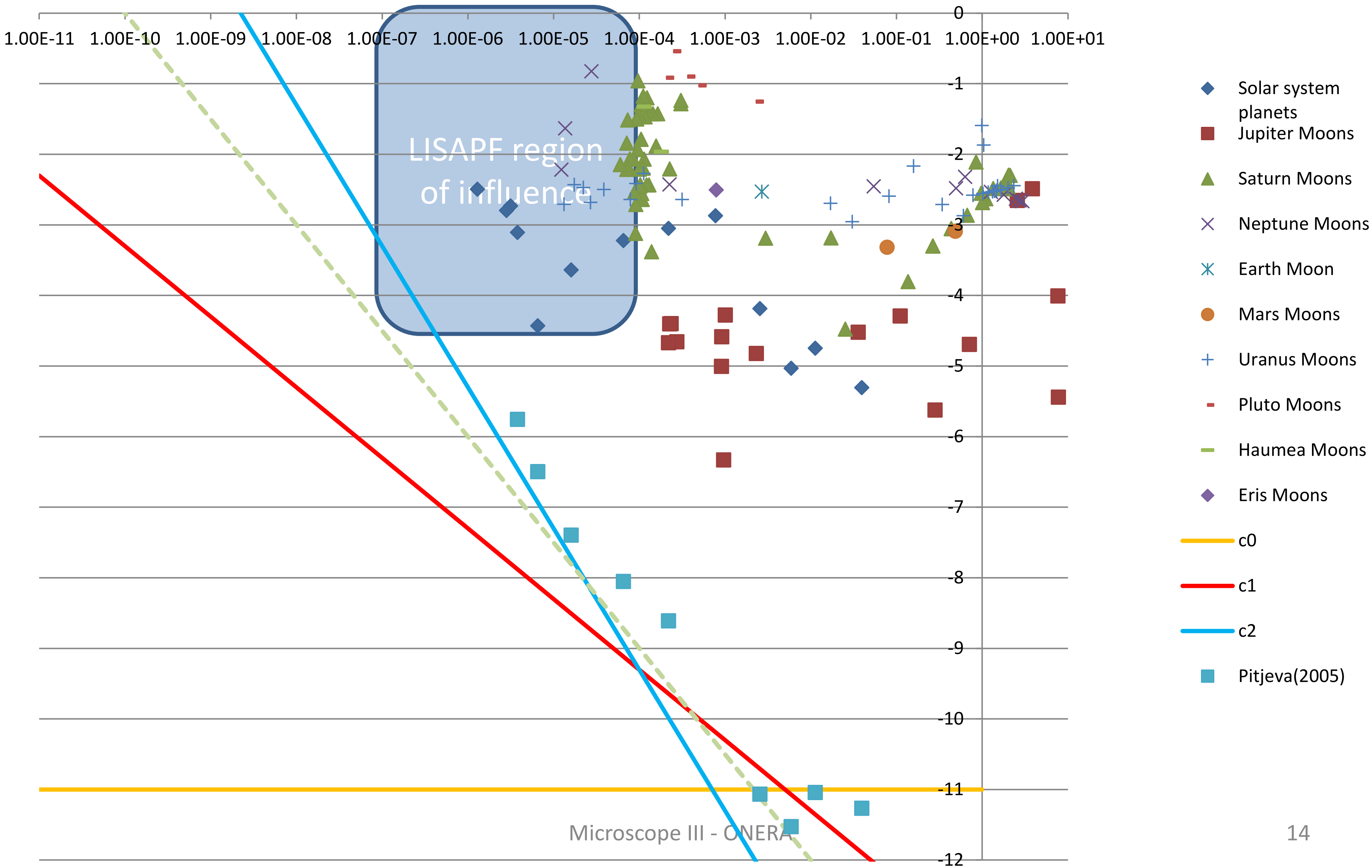


Note ephemerides line should have curvature and be lower at n=0

LISAPFU assumes $a = 5 \times 10^{-5}$
LISAPFL assumes $a = 1 \times 10^{-7}$

Need proper integration!

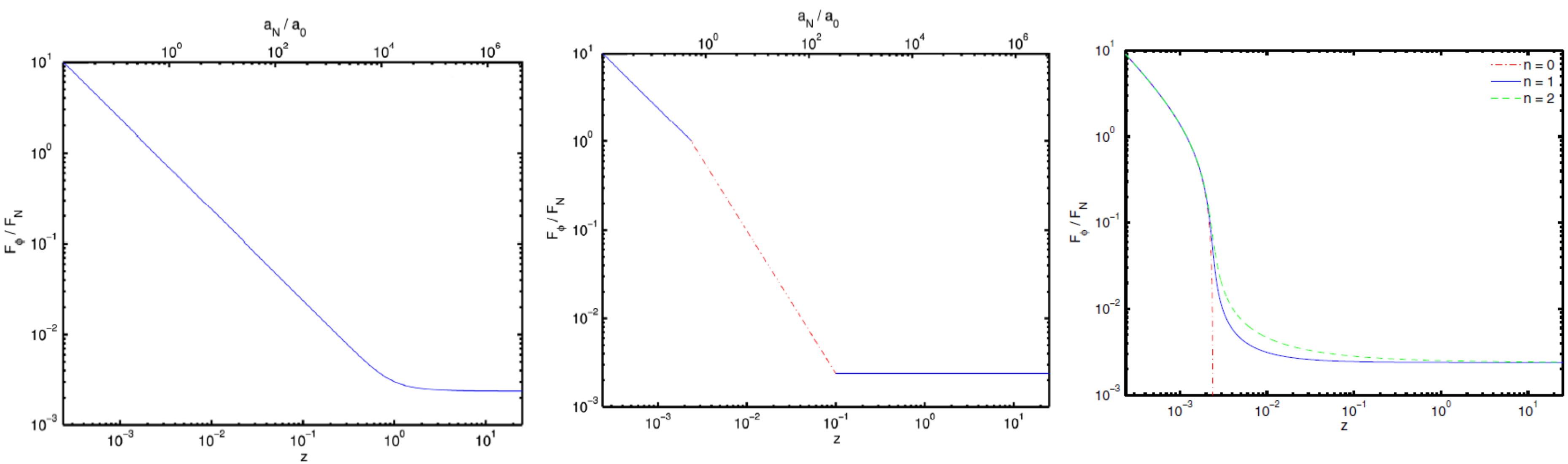
Potential LISAPF Extension



Alternative Theories of Gravity

- **Alternative Theories of Gravity:**
- **See Magueijo & Mozaffari, PRD, 85, 043527 (2012)**

  - Newtonian/Mond transition function (free function)
  - Type III can always be contrived to avoid signal



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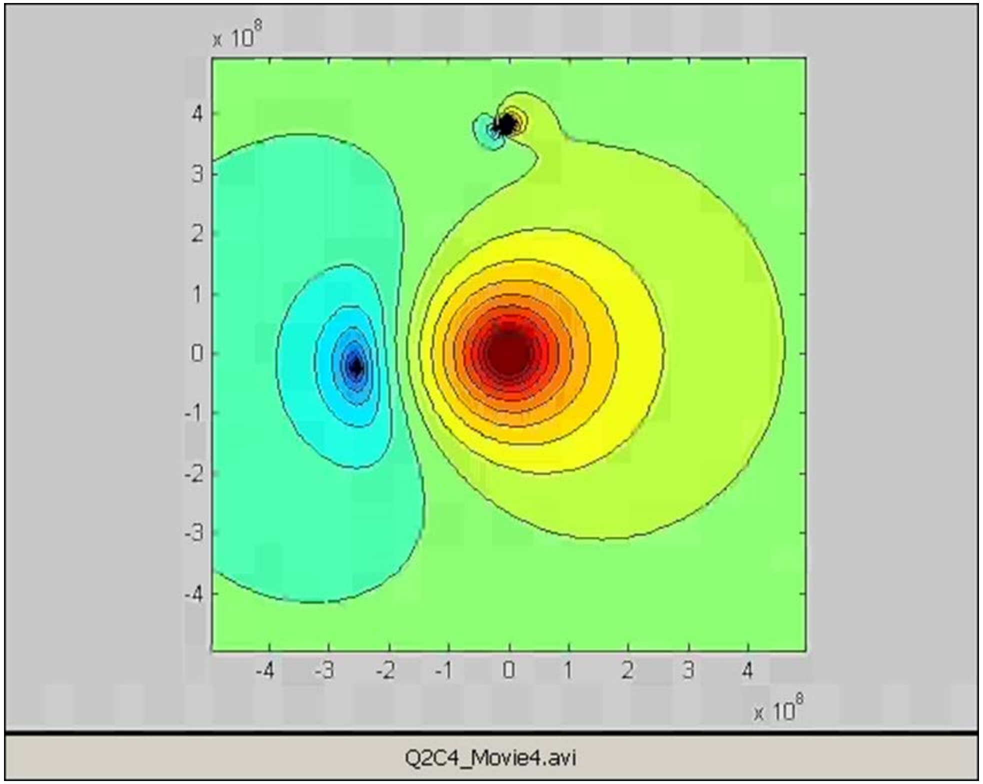
[MOND/TEVES] Tests with LISA Pathfinder

- **The “Earth-Moon SP” is perturbed by the Sun (in fact we should call it the “Sun-Moon SP”)**

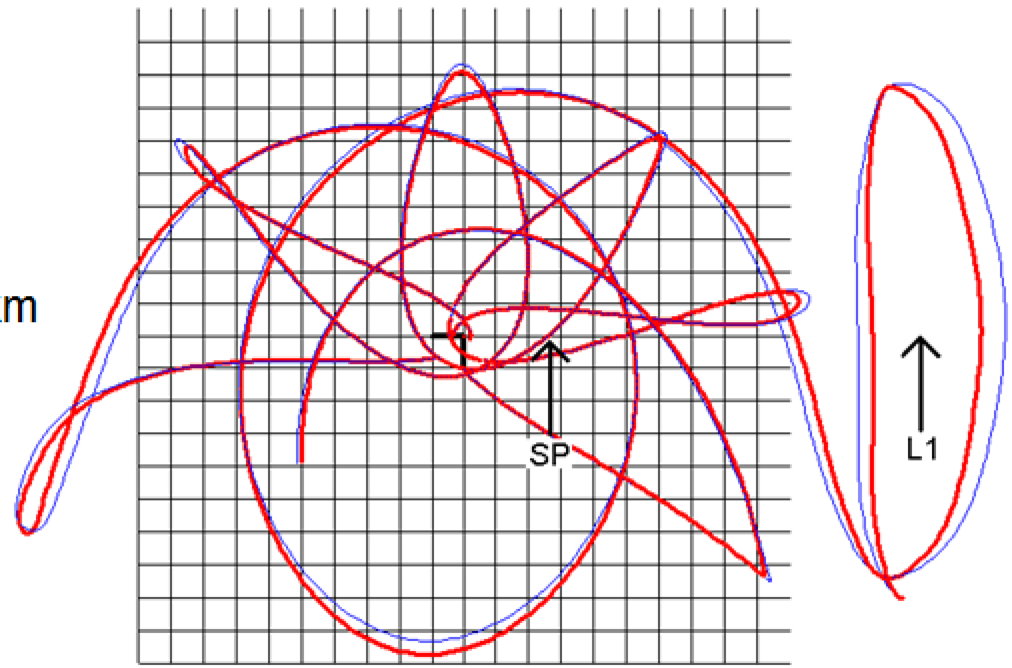


LISA Pathfinder Trajectories

- **Example of "fast" transfer:**
  - Solution transiting Sun-Earth SP within 100km

- Starting from a typical Rockot or Vega case
- Adding manoeuvre during the Lissajous orbit
- Results:
  - Miss distance around 100 km
  - Transfer time about 1 year

Vega typical start.
Transfer time: 512 days
Miss distance: 130 km



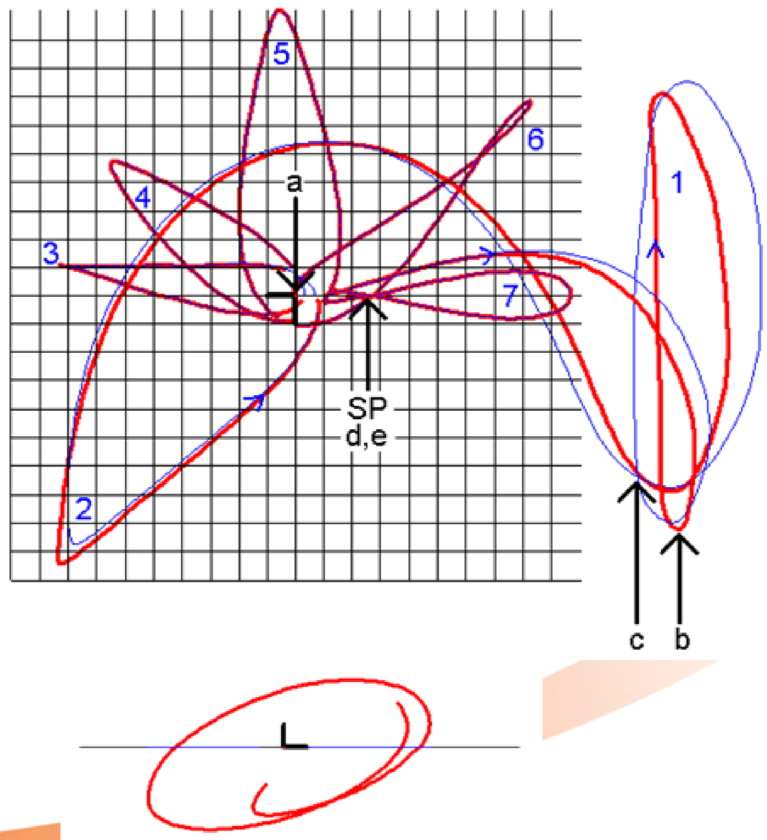
LISA Pathfinder Trajectories

- **Example of double pass:**
  - Solution achieving two SP transits by 582 days

2.2. Starting from the SP

2.2.5. First results : example

- a: Launch, 24/2/2013.
  inclination = 57.6°
  perigee altitude: 322 km
- b: Libration orbit, 73 days after launch.
- c: Exiting libration orbit, 258 days after launch. The spacecraft has spent 185 days around L1.
- d: Reaching the SP for the first time, 543 days after launch (285 days after escaping from L1).
- e: Reaching the SP for the second time, 582 days after launch (39 days after the first passage).
- Major/minor axes of the libration orbit:
  out of ecliptic: 417,000 km
  in plane: 775,000 km



Conclusions

- Limits from results of numerical integrations of orbits of solar system bodies by Pitjeva in 2005 have been recovered
- Using a model-independent approach the potential for LISAPF to make sensitive gradiometer measurements to extend the range of validity and precision has been assessed.
- The acceleration range can be extended down to 1×10^{-7}ms^{-2} .
- The precision of the validation at the low acceleration range (1×10^{-7}ms^{-2} to 5×10^{-5}ms^{-2}) can potentially be marginally improved.
- The extended range corresponds effectively to three times the current size of the solar system.
- A positive departure from Newtonian would have implications for dark matter.
- On-board propulsion allows for two passes over a few year period.
- Cost to ESA is estimated at ~10MEuro level