The Equivalence Principle and the Origin of Mass

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Newtonian Mass

In Newtonian physics, mass is a primary, undefined property of matter. Newton called it “quantity of matter”. This denomination was hiding the fundamental assumption that mass is conserved: it can be redistributed but neither created nor destroyed.

cf Lavoisier: “Rien ne se perd, rien ne se crée, tout se transforme”.

In addition, Newton assumed that mass had two roles: inertia and the power to gravitate. He noticed that experimental facts suggest that the “power of gravity” is proportional to the “quantity of matter”. The remarkable nature of this dual role was clear to him but was largely forgotten by his successors.

Einsteinian Mass: 1905

Einstein's remarkable equation $E = M\,c^2$ profoundly changed the status of mass within physics. The defining property of mass, i.e. its conservation, got undermined. Energy is conserved but (rest) mass is (a priori) not.

This opens the issue of the origin of mass: how and why can some amount of energy condense itself in a stable, localized packet of energy?

$$M = \frac{1}{c^2}\left[\int d^3x\, T^{00}\right]_{\text{rest frame}}$$

$$E^2 - c^2\,\vec{p}^{\,2} = M^2\,c^4$$

Einsteinian Mass: 1915

As early as 1907, the second role of mass (linked to gravity) led Einstein on a path towards a new theory of gravitation where the source of gravitation would be the energy.

Using the equivalence principle (EP), i.e. the universality of free fall, as a heuristic tool he postulated that the souce of gravity must be the stress-energy tensor $T^{\mu\nu}(x)$ which is the fullest description of energy (and energy conservation: $\partial_\nu T^{\mu\nu} = 0$) in Special Relativity. As shown by him (and later by others, starting from the consistency of a coupling $h_{\mu\nu} T^{\mu\nu}$ generalizing $A_\mu J^\mu$) this leads (quasi-)uniquely to General Relativity (GR):

$$R_{\mu\nu} - \frac{1}{2} R\, g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

Quantum Field Theory (QFT) Mass $\geq$ 1930

Relativistic QFT opened new vistas on the concept mass.

- Quantum fields can create and destroy particles. Massive particles can decay into other massive particles, if $m_0 \geq m_1 + m_2$
- Massless particles (satisfying $E^2 - c^2 \overrightarrow{p}^2 = 0$) are on par with massive ones (satisfying $E^2 - c^2 \overrightarrow{p}^2 = m^2 c^4$) and, actually, look simpler and possibly more fundamental (e.g. photon, graviton)
- Mass and Spin appear as parameters classifying irreducible, unitary (quantum) representations of the Poincaré group (Wigner 1939)

Scale-dependent Mass in QFT

• In classical field theory (Lorentz, Abraham, . . ., Dirac, . . .)

$$\text{electron mass :} \qquad m(r) = m_0 - \frac{e^2}{2r}$$

additive renormalization which includes the field energy around the particle.

• In QFT the answer is surprisingly different for elementary particles

$$\text{electron (and other spin } \tfrac{1}{2} \text{ fermions)} \quad m(\Lambda) = m_0 \left[1 + \frac{3\,e^2}{4\,\pi} \left(\ln \frac{\Lambda^2}{m_0^2} + \frac{1}{2} \right) \right]$$

multiplicative and logarithmic renormalization

$$\text{scalar :} \qquad m^2(\Lambda) = m_0^2 + c\,\Lambda^2$$

additive, quadratic renormalization.

Dimensional transmutation: "mass without mass"

- Classically scale-free nonabelian gauge theory (gluons)
- Quantum-mechanical mass scale $\Lambda_{\rm QCD}$ where $g_{\rm SU(3)}(\Lambda_{\rm QCD}) \sim 1$
- $\Lambda_{\rm QCD}$ essentially determines the mass of all ordinary objects

$$m_p \simeq m_n = c\,\Lambda_{\rm QCD}$$

$$m_{\rm atom} \simeq (N+Z)\,c\,\Lambda_{\rm QCD}\left[1 + \text{corrections} \propto e^2, \frac{m_u}{\Lambda_{\rm QCD}}, \frac{m_d}{\Lambda_{\rm QCD}}, \frac{m_e}{\Lambda_{\rm QCD}}\right]$$

where

$$m_u = Y_u \langle H\rangle \quad \text{with} \quad H\text{-boson VEV } \langle H\rangle \sim \frac{\mu}{\sqrt{\lambda}}$$

H-mass scale μ only explicit mass scale in SM Lagrangian

An invisible but important actor: the "vacuum"

- In classical physics, "empty space" is a ghost which, however, acts on matter through geometry and inertial properties (Newton's pail)
- In GR matter acts on space by deforming its structure
- In QFT the vacuum is not only full of fluctuating fields and particles, but it is filled with condensates that have directly observable physical properties: quark condensate $\langle \bar{q}\, q \rangle$, H boson condensate $\langle H \rangle$, geometry condensate $\langle g_{\mu\nu} \rangle$
- The vacuum has many connections with "mass"
  - mass of hadrons from "bags" within a perfect color dia-electric vacuum (color-electric Meissner effect of vacuum)
  - mass of leptons from $\overline{\psi}\, H\, \psi$ coupling of initially massless fermions

More connections between mass and the vacuum

- Astrophysics and cosmology show that most of the mass in the Universe is $\sim 26\%$ dark mass and $\sim 70\%$ dark energy
- Quantum fluctuations and the mystery of dark energy

$$\rho_{\rm vac} = \frac{\hbar}{2} \sum_i (-)^{F_i} \, \omega_i = \frac{\hbar}{2} \sum_i \int \frac{d^3 k}{(2\pi)^3} (-)^{F_i} \sqrt{m_i^2 + \mathbf{k}_i^2}$$

? need many cancellations (Pauli: $\sum (-)^{F_i} = 0$, $\sum (-)^{F_i} m_i^2 = 0$, $\sum (-)^{F_i} m_i^4 = 0, \ldots$) to end up with a very small $\rho_{\rm vac}$

$$\rho_{\rm vac} \simeq (2.3 \times 10^{-3} \, eV)^4 \simeq 10^{-123} \, M_{\rm Plack}^4$$

Mass and the Equivalence Principle

(i) The more we understand mass, the more complex it looks: the mass of an atom comes mostly from $\Lambda_{\mathrm{QCD}} \simeq \Lambda_{uv}\, e^{-c/g_{uv}^2}$ (dimensional transmutation), with non universal corrections coming from QED (e^2), quark masses $(m_u = Y_u \langle H \rangle, \ldots)$ and the electron mass $(m_e = Y_e \langle H \rangle)$.

(ii) The EP asserts that gravity is totally blind to the complexity of the structure of mass. This appears even more remarkable today than at the times of Newton or Einstein. Modern physics would easily allow for many possible types of EP violation because it allows for possible couplings to the various elements entering the complex structure of mass.

(iii) In a similar vein: the energy of the vacuum became an issue only after one realized the complicated structure hidden in "empty space".

(iv) Similarly to the possible anthropic explanation of $\rho_{\mathrm{vac}} \simeq 10^{-123}\, M_{\mathrm{Planck}}^4 \neq 0$ (Weinberg), one can imagine that there will be EP violations at the maximal level compatible with the development of life (Damour-Donoghue 2010).